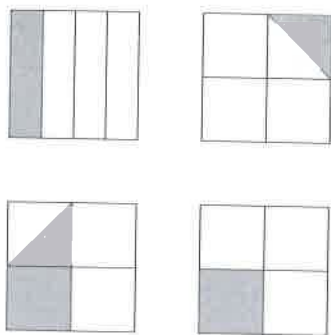# AMC 10/12 Student Practice Questions

Guide to Student Practice Questions

Each of the following four large congruent squares is subdivided into combinations of congruent triangles or rectangles and is partially shaded. What percent of the total area is partially shaded?

The original problem and choices from the 2011 AMC 8 contest

(A) $12\frac{1}{2}$ (B) 20 (C) 25 (D) $33\frac{1}{3}$ (E) $37\frac{1}{2}$

2011 AMC 8, Problem #7—
"Find the shaded portion of each square separately."

Problem number
Hint

Solution

Answer (C): The upper left and the lower right squares are each one-fourth shaded, for a total of one-half square. The shaded portions of the upper right and lower left squares make up one-half square. So the total shaded area is one full square, which is 25% of the total area.

Solution from official solutions

Difficulty: Medium
SMP-CCSS: 2, 7
CCSS-M: 6G.1, 6RP.3C

Standards for Math Practice
Common Core State Standard

Difficulty, Percent correct

Easy	100-80%
Med Easy	80-60%
Medium	60-40%
Med Hard	40-20%
Hard	20-0%

Cagney can frost a cupcake every 20 seconds and Lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?

(A) 10   **(B)** 15   **(C)** 20   **(D)** 25   **(E)** 30

2012 AMC 10A, Problem #1—
2012 AMC 12A, Problem #2—

"Find how many cupcakes Cagney and Lacey can make individually in five minutes."

Solution

Answer (D):   Because 20 seconds is $\frac{1}{3}$ of a minute, Cagney can frost $5 \div \frac{1}{3} = 15$ cupcakes in five minutes. Because 30 seconds is $\frac{1}{2}$ of a minute, Lacey can frost $5 \div \frac{1}{2} = 10$ cupcakes in five minutes. Altogether they can frost $15 + 10 = 25$ cupcakes in five minutes.

Difficulty: 10: Easy
12: Easy
SMP-CCSS: 2
CCSS-M: N-Q.1

Let $\angle ABC = 24°$ and $\angle ABD = 20°$. What is the smallest possible degree measure for $\angle CBD$?

(A) 0   **(B)** 2   **(C)** 4   **(D)** 6   **(E)** 12

2012 AMC 10A, Problem #4—

"Consider the fact that AB is a common ray to both angles."

Solution

Answer (C):   Ray AB is common to both angles, so the degree measure of $\angle CBD$ is either $24 + 20 = 44$ or $24 - 20 = 4$. The smallest possible degree measure is 4.

Difficulty: Medium Easy
SMP-CCSS: 1, 5
CCSS-M: 7.G5

The product of two positive numbers is 9. The reciprocal of one of these numbers is 4 times the reciprocal of the other number. What is the sum of the two numbers?

(A) $\dfrac{10}{3}$ (B) $\dfrac{20}{3}$ (C) 7 (D) $\dfrac{15}{2}$ (E) 8

2012 AMC 10A, Problem #6—

"Denote each number as a variable and create a system of equations that correspond to their relationships."

Solution

Answer (D): Let $x > 0$ be the first number, and let $y > 0$ be the second number. The first statement implies $xy = 9$. The second statement implies $\frac{1}{x} = \frac{4}{y}$, so $y = 4x$. Substitution yields $x \cdot (4x) = 9$, so $x = \sqrt{\frac{9}{4}} = \frac{3}{2}$. Therefore $x + y = \frac{3}{2} + 4 \cdot \frac{3}{2} = \frac{15}{2}$.

Difficulty: Medium
SMP-CCSS: 2
CCSS-M: 8.EE8b

Mary divides a circle into 12 sectors. The central angles of these sectors, measured in degrees, are all integers and they form an arithmetic sequence. What is the degree measure of the smallest possible sector angle?

(A) 5   **(B)** 6   **(C)** 8   **(D)** 10   **(E)** 12

2012 AMC 10A, Problem #10—
2012 AMC 12A, Problem #7—

"Find the sum of the degrees of the central angles in terms of the initial term and common diffrence."

Solution

Answer **(C)**:   Let a be the initial term and d the common difference for the arithmetic sequence. Then the sum of the degree measures of the central angles is

$$a + (a + d) + \cdots + (a + 11d) = 12a + 66d = 360,$$

so $2a + 11d = 60$. Letting $d = 4$ yields the smallest possible positive integer value for a, namely $a = 8$.

Difficulty: 10: Medium Hard
12: Medium Hard
SMP-CCSS: 2, 7
CCSS-M: G-C.2

It takes Clea 60 seconds to walk down an escalator when it is not operating, and only 24 seconds to walk down the escalator when it is operating. How many seconds does it take Clea to ride down the operating escalator when she just stands on it?

(A) 36   **(B)** 40   **(C)** 42   **(D)** 48   **(E)** 52

2012 AMC 10B, Problem #13—
2012 AMC 12B, Problem #9—

"Assign variables to the rate of walking and the rate of the moving escalator, then compare times and rates for the same distance."

Solution

Answer (B):   Let x be Clea's rate of walking and r be the rate of the moving escalator. Because the distance is constant, $24(x + r) = 60x$. Solving for r yields $r = \frac{3}{2}x$. Let t be the time required for Clea to make the escalator trip while just standing on it. Then $rt = 60x$, so $\frac{3}{2}xt = 60x$. Therefore $t = 40$ seconds.

Difficulty: 10: Medium
12: Medium
SMP-CCSS: 4
CCSS-M: A-CED.2, A-REI.6

Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

(A) $\dfrac{1}{8}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{\sqrt{10}}{10}$ (D) $\dfrac{\sqrt{5}}{6}$ (E) $\dfrac{\sqrt{10}}{5}$

2012 AMC 10B, Problem #17—
2012 AMC 12B, Problem #15—

"Find the slant heights and circumferences of the bases of the cones.'

Solution

Answer (C): Each sector forms a cone with slant height 12. The circumference of the base of the smaller cone is $\frac{120}{360} \cdot 2 \cdot 12 \cdot \pi = 8\pi$. Hence the radius of the base of the smaller cone is 4 and its height is $\sqrt{12^2 - 4^2} = 8\sqrt{2}$. Similarly, the circumference of the base of the larger cone is 16π. Hence the radius of the base of the larger cone is 8 and its height is $4\sqrt{5}$. The ratio of the volume of the smaller cone to the volume of larger cone is

$$\frac{\frac{1}{3}\pi \cdot 4^2 \cdot 8\sqrt{2}}{\frac{1}{3}\pi \cdot 8^2 \cdot 4\sqrt{5}} = \frac{\sqrt{10}}{10}.$$

Difficulty: 10: Hard
12: Medium Hard
SMP-CCSS: 4, 5
CCSS-M: G-GMD.4

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